Mathematical saturation within workplace contexts

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In this paper I propose that the literature on situated numeracy has an important role in the reconceptualisation of mathematics education in general and workplace learning in particular. There exists a substantive literature which documents the idiosyncratic knowledge and procedures used by participants *in situ* to resolve tasks. This literature has been powerful in challenging the orthodoxies within the field of mathematics education whereby the power of mathematics comes from its capacity to be applied across a wide range of contexts. However, this situatedness of mathematics must be considered in conjunction with the degree of mathematics employed within a context. To this end, I propose and develop the notion of "mathematical saturation" which permits the breaking of the dichotomy of school-mathematics and non-school mathematics, and in its place proposes a continuum in which the degree of "pure" mathematics is considered to be a critical element. Using the data collected from a number of worksites, it will be argued that situated numeracy challenges notions of transferability but must be considered alongside the degree of formal mathematics which is used within that context.

Situated Numeracy

Within the field of mathematics education, challenges have been proposed which destabilise the orthodoxy guiding significant practices and discourses. These challenges have come from the discourses which support notions of numeracy being situational and hence highly determined by and determining the context within which the mathematics is used. Such challenges have had significant impact in the theorisation of the role and purposes of mathematics in out-of-school contexts. As such, there are important implications for teaching and learning practices when considered in conjunction with the current discourses on key competencies.

Over the past decade, the work broadly considered "situated numeracy" has brought to the fore the processes, knowledge and skills used in non-school contexts and has effectively challenged the assumption that mathematical knowledge and skills can transfer from school to non-school contexts. In contrast, situated numeracy has highlighted the highly contextual strategies used by individuals solving everyday tasks. The mathematics and problem solving strategies used in everyday contexts are substantially different from those of the formal school mathematics and in most cases, are often far more effective. This literature has posed serious challenges to the approaches underpinning much of contemporary mathematics education. The literature on situated numeracy has had a significant impact in mathematics education since it challenges many of the dominant myths and assumptions that have underpinned significant components of mathematics. Two main areas of challenge are discussed.

First is the transferability of knowledge from one context to another. One of the more persuasive assumptions underpinning mathematics education is the notion that the mathematical thinking and reasoning embedded in school mathematics can be transferred to non-school contexts. This is a powerful belief and supports the status of mathematics within the formal school context and in the wider social arena. This position supports the notion that abstract concepts and operations learned with the school context can be transposed across other contexts. For example, if the student learns the basic operation of addition, eg 3+4=7, then this knowledge will be useful in contexts such as shopping (3 apples + 4 bananas = 7 fruit), fuel consumption (3 litres + 4 litres = 7 litres) or workplaces (3 metres of 4x2 hardwood + 4 metres of 4x2 hardwood = 7 metres of 4x2 hardwood). Lave (1988) and Walkerdine (1988) have challenged such positions by proposing that the everyday mathematical processes used in resolving what appear to be mathematical tasks are substantially different from those undertaken in the formal school context. Lave's (1998) study of shoppers has shown that processes of checking prices on cheeses does not align itself with the ratio problems taught in the school-context. Kanes (1996) study of motel check-in staff shows that the simple addition tasks involved in calculating accounts are influenced by the motel goods consumed. Similarly, the study of Brazilian street vendors (Carraher, 1988; Carraher, Carraher, & Schliemann, 1985) and candy sellers (Saxe, 1988) offer strength to such proposals. Through the research into everyday practices, the question as to whether knowledge does transfer from one context to another, and in particular whether the mathematical knowledge learned within the school context transfers to the everyday context, is challenged.

Second, is subsequent to this challenge. If indeed, mathematics is not transferable, then the status of mathematics is under threat. The status of academic knowledge over practical, everyday knowledge is threatened. School mathematics is high-status knowledge which maintains it power through assumptions that the knowledge is powerful because it is abstract, transcultural and transposable. The situated numeracy discourses raise challenges to these beliefs.

The move towards those situated learning approaches in which the learning and the social context are inextricably bound together so as to be mutually constitutive is useful in demonstrating the power of the context in determining the processes and knowledge used in task resolution. The skills used are often distinctly different from those learned in school maths, and moreover are more effective. Carraher et al's studies have shown the efficacy of situated numeracy and the inefficacy of school-maths. The activity which is undertaken within the workplace must be understood as a dialectical engagement: it is the self-organising and reciprocal arrangement between socially-constituted people and socially-constituted settings. Within such an approach, the goals commonly associated with school mathematics, such as abstraction and deductive reasoning, are not the same goals of the workplace, so that the forms of mathematics are different.

In studies of out-of-school contexts, research has valorised the practical knowledge and understandings commonly associated with "common sense". The Carraher et el studies indicate, for example, that the mathematising of practices often confuses students. In contrast, young students in their everyday contexts are able to conduct efficiently and effectively marketplace calculations but in the school context are unable to complete similar calculations. These studies suggest that the differentiation between the two practices and the legitimacy and status of school knowledge may be inappropriate. The implications of this research is the challenge to school mathematics to become more vocationally orientated and that since the context is profoundly interwoven with cognition, then learning should occur in contexts most like those to which the students will graduate. This has substantive implications for workplace reform in concert with school-maths reforms.

Applications and Situated Numeracy

In contrast to this research is the work founded on the notion that adopting an approach which uses everyday problems is restrictive to the development of substantive mathematical reasoning. Within this approach, "pure" mathematics is assumed that the more decontextualised the mathematics, the more likelihood there is of transfer. Sierpinska (1995, p.2) considers the implications of this work for school maths practices and argues for a position somewhere between the two extremes. She suggests that in order to be able to apply decontextualised mathematics to practical, everyday situations, students must be "taught the art of applications" (p. 3). She goes on to argue that it is not the distinction between the everyday and the school contexts which marks the difference but in the nature of the problems being given. In the everyday context, there is an authentic problem which requires the students to employ problem posing strategies such as those suggested by Silver (1994). In this problem posing context, authentic problems are generated by the students and in doing so, they are free to choose resolution strategies rather than being bounded by prescribed rules and procedures. Sierpinska (1995) argues that in order to complete the tasks within a context, particular rules of logic are called to task. In the case of school mathematics, very particularised practices are employed which have been learned over a period of time. In contrast, the methods used within the everyday context are substantially different, often not calling on even the simple mathematics of school. In order to complete the everyday tasks, Sierpinska proposes that the individual is "not so advanced in her mathematics. To get to this point she must first learn simple school arithmetic" (p. 5).

The differences between a workplace or non-school environment and a more traditional "school" environment can be observed in the forms and processes of mathematics which are employed by participants. Whereas formal school contexts adopt problem solving as an approach which offers considerable advantages over the more traditional "chalk and talk" teaching methods, the problem solving found in the workplace is substantially different from that of the school context. Workplace problem solving demands that learners apply situational mathematical knowledge and concepts to situations which are constrained by real-life problems. For example, Masingila (1993) suggests that her study of carpet laying requires not only the concept of area to be applied, but also real life constraints applied to the laying context which include the nap of the carpet and placement of seams. Masingila's (1993) studies of carpet layers lead her to suggest that the situated mathematics of this group of people is different from school mathematics in a number of key dimensions.

- The people are actively engaged in DOING mathematics. The constraints of their jobs pose REAL problems which must be solved.
- The apprenticeship model of learning provides a good method for teaching/learning the occupation.
- Textbooks could not provide the detailed constraints of the occupation which would hinder the problem-solving skills needed for the job.
- Learning-in-context creates a broader learning of a concept due to the active involvement in real problem solving tasks demanded by the job.

The issue which needs to be considered is whether school-based tasks actually represent the real life contexts which they are purported to do, or whether they are a veneer which serves to support the legitimacy of mathematics as a discipline which can be reappropriated and applied across many contexts. Masingila (1993) suggests that the complexity of problem solving demanded in certain occupations could not be represented in a text-book task and that often what it represented in such textbooks is a computational task masked in an everyday situation. The resolution of such tasks is dissimilar to the processes and skills that would be employed by a worker working with a real problem. Often the processes which are taught in formal contexts such as schools often prevent or discourage the development of links between workplace and school mathematics.

Situated learning has offered improved understandings of the cognitive processes used in the resolution of everyday tasks and challenged the myths of transferability of knowledge from the school context to the everyday, practical context. Sierpinska has raised the challenge to consider the authenticity of tasks - school and everyday - and the consequent mathematical skills and knowledge used in that context. While Lave's shoppers were very effective in solving the pricing of cheeses, the strategy may not have been as effective had a mathematical strategy been employed. Quite clearly there are situations which call for highly mathematical methods for resolution while others can be resolved through everyday methods which are not so heavily reliant on formal mathematical methods. To this end, the mathematics employed within a context influences and is influenced by the particular demands of the context. What is proposed in the remainder of this paper is the notion of mathematical saturation.

Mathematical Saturation

In the following sections, I develop the notion of mathematical saturation through the use of three distinct workplace environments. In each context, there are distinctly different forms of mathematics used and demanded. The choices of mathematics available to a participant are constrained by the background knowledge of the student thus lending support to Sierpinska's (1995) earlier comments about the need to be mathematically numerate to be able to operate effectively and mathematically.

In developing the notion of mathematical saturation, "mathematics" is seen to be the pure form of mathematics used in the formal school context. This is in line with Dowling's (1997) proposal of mathematical saturation in mathematics texts. Rather than propose a model in which there are, or are not, modes of mathematics operating (as in many of the approaches to situated numeracy); or that there are models in which pure mathematics is seen to be the most appropriate strategy resolution; or somewhere in the middle as proposed by Sierpinska, this model suggests that there is a continuum which spans from contexts where the amount of school mathematics is negligible through to contexts which are highly mathematical.

The following examples are taken from a research project in which employees were interviewed and then followed for a period of time by the researcher. The interviews consisted of ascertaining the employees conceptualisation of their work duties and what, if any, mathematics they used in their employ. A range of industries were involved in the project but only those involved in fast-foods, construction and nursing will be reported on in this paper.

Low Saturation- Crew member at MacDonalds

Contexts where there is little need for the application of formal mathematics to resolve the task can be seen to be low in saturation. In these contexts, the participant develops effective and efficient strategies for task resolution without embedding strong mathematical knowledge and skills. For example, the choices made by shoppers as to whether to purchase particular sized items based on constraints such as whether the containers will fit in the cupboards or whether the goods would be consumed within the stay-fresh period indicate low mathematical saturation. Clearly the problem solving strategies are highly contextual but they are low in mathematical content. Such situations would be solved in the formal mathematics contexts as ratio problems.

Crew members at MacDonalds are required to prepare and stock the counters. Most of the tasks are automated with very little autonomy for the employees. In spite of the need to measure time, quantities, money and estimation, most of these tasks are automated. For example, cooking pancakes requires the employee to push a handle and the contents are exuded on the griller in set amounts of set size. A timer sounds when the pancakes require turning. After turning and cooking the reverse side, the pancakes are then placed in the tray.

The employee comments aptly sum up the mathematical saturation in the workplace:

> Crew: Don't really use maths, just do it as MacDonalds says because it has to be the same all the time. You learn how they want it done and do it like that.

An examination of the workplace skills needed indicate some levels of mathematics but these are minimal. Handling money is highly electronic in that totals are calculated according the items pressed (rather than amount) and change calculated. Some counting skills are necessary to ensure the correct change is taken from the cash register. Similarly, in preparing the mixture for the pancakes, large packets of premix are required to be mixed with particular amounts of water. This water must be measured and then mixed into the prepared to mix to form the batter. More generic skills of problem solving are needed to ensure that there is a constant flow of food, but these skills are not necessarily the responsibility of the junior crew member.

When considering the context of the MacDonalds crew member, the situated numeracy is minimal and the degree of mathematics embedded in the tasks to be undertaken was very low. Most of the mathematics has become automated so that the need to quantify or measure was almost eliminated. Minimal counting was required – as in the water:pancake mix ratio – so that there was very little mathematical saturation.

Medium Saturation - Construction Work

In the second scenario, case studies of construction workers were undertaken. The study of pool builders provided evidence of highly contextual numeracy and problem solving with some use of formal school mathematics *albeit* relatively limited. It is therefore proposed that medium saturation was evident in the use of mathematics in this context, but it was not necessarily of the formal school mathematics. Mathematics was used, but it was modified for and by the context. The type/s of mathematics used varied and depended on the tasks being undertaken.

Various teams are used in the construction of pools which can be broadly conceived of as excavators whose task is to prepare the site for the pool including the excavation of the pool hole; box-and-framers whose task is to construct the free-standing frame for the pool; the concreters who are responsible for the formation of the pool; the finishers who apply the required finish (tiles, pebbles, sealer) as selected by the owner; and the minor roles of plumbers, electricians and supervisors of the whole project. In all phases of the building process, estimation skills feature strongly for all teams - estimating the amount of soil to be removed and the number of trucks needed to remove the dirt; estimating the amount of concrete needed and the application of that concrete. The estimation skills of most of the teams are well developed but are not heavily reliant on formal mathematics. Of particular importance in considering mathematical saturation are the skills of the box-and-framer who requires a mix of highly contextual numeracy skills alongside a number of formal mathematical skills.

The mathematical skills of the box-and-framer are developed and constrained by the context. The site most often is a large hole in the ground covered with a layer of screenings on the floor to allow for movement once the pool has been poured. For the box-and-framer to have access to measuring tools to mark out the positions of reinforcing bars in the box-and-frame stage would be constraining and clumsy. The framing of the pool is a three-dimensional project in which horizontal and vertical framing bars must be considered in conjunction with the flooring grid so that a vertical bar on the side of the pool will also form the flooring grid. All of these bars must run in parallel lines of equal spacing. In order to construct the frame for the pool, the box-and-framer needed to cut and bend the reinforcing bars at the appropriate places. Only the length of the bars for cutting were measured, but where bends were to be located were estimated visually. These bends needed to consider the 3 dimensions of the pool so that a bar needed to traverse the length of the pool, up the side of the pool and then splayed around the lip to provide appropriate spacing. Further more, in line with Lave's notions of experts and novices, the master box-and-framer is able to accurate estimate the placement of horizontal and vertical bars through the three-dimensional frame. For example, the 65 bars around the circumference of the pool must be positioned within 30 cm +/- 0.5 cms to comply with council regulations. There were only 5 bars which were marginally out of this range. Similarly, the placement of the lip of the pool must be 9 inches which was measured using hand spans.

The estimation skills used for measurement in this context were highly contextual and well developed. The accuracy was very high. Motivation to be accurate was high as the various phases in the construction process were sub-contracted out. Each subcontractor was responsible for his costs and hence profits. The constraints imposed by the local councils who inspected the pool at various stages of constructed also provided motivators since the pool construction had to conform with pool regulations if it were to be approved. Hence, the box-and-framer was motivated for accuracy not only to comply with building regulations, but also profits. In this context, the numeracy was highly situated and effective having developed out of the context. The box-and-framer had an extremely high degree of accuracy without relying on formal mathematics.

High Saturation - Nursing

As with other professions, nurses have developed numeracy skills which are situational. A number of aspects of nurses' work require various degrees of mathematics. Many of the traditional tasks requiring mathematical calculations have become redundant. For example, intravenous drips have been modified so that the tedious calculations for drip rates have been replaced with a type of sliding scale found on the IVAC pumps. This allows the nurse to enter the amount of saline drip to be consumed within the time nominated so eliminating calculations.

Where mathematics has become highly situational, nurses have developed idiosyncratic ways of documenting aspects of their work. This documentation is a unique component of the medical profession where not only has accountability become greater, but economic rationalism has meant that more patients need to be tended to in smaller amounts of time so quicker methods of patient assessment must be undertaken. Having patient information readily accessible in some format for doctors and changeovers means less patient-medico time. This documentation of patient-responding-to-treatment can be observed in wards where there is the need to monitor wound healing rates, such as ulcerated limbs. The nurse estimates the length, width and depth of the wound and then maps this onto a graph. Changes are noted over time, and the changes noted on formal documents. Similarly, in the obstetric wards, newborns are given an AGPAR score to indicate the overall health status of the child. This is an almost immediate score which consists of ratings which are on a 0, 1 or 2 scale which are then summated to give an overall score out of ten. This is then calculated again after a short period of time to ensure that the child is maintaining or improving in health status. This score is often an immediate response, rather than the mathematical task of school mathematics. There is minimal mathematics embedded in the tasks, but rather "professional judgement" is seen to be more important. There is no formal measurement of wounds or infant colour, but rather the judgement of the nurse is the formal documentation. Nurses were very confident in this aspect of their work.

One aspect of nursing containing a high level of mathematical saturation is problematic for many nurses. Unlike other aspects of the profession where situated numeracy skills can be used and developed, the ratio problems of drug consumption are highly mathematical. For example, in post-operative care, the patient may be required to consume Heparin which is a drug used as an anticoagulant in order to prevent DVT (deep vein thrombosis). The doctor may prescribe a dosage of 3000 units per ml but it is stocked at a strength of 5000 units per ml. In order to give the patient the correct dosage, precise calculation needs to be undertaken. Over-dosage may result in death so motivation should be considered as extremely high. As in other contexts, motivation provides a strong context for ensuring "correct" resolution of tasks yet in this context, the mathematics subsumes the meaningfulness of the task and hence is often undertaken as a meaningless operation as noted in the comment made be a nurse:

> Nurse: To work out what I have to give, I get "strength required' over "strength in stock" multiplied by volume. If I have to work out a Hep [Heparin] dose, I would have 3000 over 5000 multiplied by 1 ml. That gives me 0.6 ml.
>
> R: How did you work that out?
>
> Nurse: It is one we do all the time, so I just know it. I get a bit confused with the way to do it if I get a new one. I have to think about which number is on the top and which is on the bottom.

The lack of confidence in understanding the actual processes involved in working through the problem indicates the nurse's conceptual understanding of what is actual being undertaken. The high degree of reliance on formulae is reminiscent of school mathematics. Unlike the box-and-framer where motivation is closely linked to profit and loss, nurse's calculations are often life and death situations so it would be expected that there would be higher motivation in this context and hence greater motivation for task resolution. Yet as nurse educators and hospital administrators are acutely aware, this aspect of nurse professional development is critical. yet the degree of mathematical saturation hinders nurses' capacity to effectively undertake the task.

Conclusion

The types of mathematics used and developed within a context appears to have a significant bearing on the confidence of the participants and their subsequent efficacy. Where there is high degree of mathematical saturation, there is less chance of the participants feeling confident and empowered with that situation. Indeed, as nurses have reported, they are not confident in using the mathematics for calculating ratios for drug use in spite of its high degree of motivation - being a life and death situation. In contrast, where the level of mathematical saturation is substantially lower and the mathematics has been developed within a context such as the box-and-framer's placement of bars and the nurses' AGPAR scores, there is a great sense of confidence in the results.

The notion of mathematical saturation needs to be developed further as it has merit in the context of situated numeracy. From the examples used in this paper, there are varying degrees of mathematics embedded in the workplaces observed. In each setting, task motivation varies along with the degree of mathematics being employed. What becomes clearer is what is considered to be "mathematics" and how this then becomes conceptualised as situated numeracy is of great importance in this area of adult and workplace learning.

References

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more real? *For the Learning of Mathematics, 13*(2), 12-17.

Carraher, T. N. (1988). Street mathematics and school mathematics. In A. Borbas (Eds.), *Proceeding of the twelfth PME conference* (pp. 1-23). Veszprem, Hungary: International group for the Psychology of Mathematics Education.

Carraher, T. N., Carraher, D. W. &., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology, 3*, 21-29.

Carraher, T. N., Carraher, D. W. &., & Schliemann, A. D. (1987). Written and oral mathematics. *Journal for Research in Mathematics Education, 18*(2), 83-97.

Damarin, S. K. (1993). Schooling and situated knowledge: Travel or tourism. *Educational Technology, 33*(3), 27-32.

Dowling, P. (1997). *A sociological analysis of mathematics.* London: The Falmer Press.

Helme, S. (1995). Maths embedded in context: How do students respond? *Numeracy in Focus, 1*(Jan), 24-32.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* Cambridge: Cambridge University Press.

Lave, J. (1992). Learning as participation in communities of practice. Paper presented at the *American Educational Research Association, Conference*, San Fransisco. April.

Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate peripheral participation.* Cambridge: Cambridge University Press.

Lave, J., Murtaugh, M., & de la Rocha, O. (1984). The dialectic of arithmetic in grocery shopping. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context.* (pp. 67-94). Cambridge: Cambridge University Press.

Masingila, J. O. (1993). Learning from mathematics practice in out-of-school contexts. *For the Learning of Mathematics, 13*(2), 18-22.

Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher, 17*(6), 14-21.

Sierpinska, A. (1995). Mathematics: "in context", "Pure", or "with applications"? A contribution to the questions of transfer in the learning of mathematics. *For the Learning of Mathematics, 15*(1), 2-15.

Silver, E. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.

Walkerdine, V. (1982). From context to text: A psychosemiotic approach to abstract thought. In M. Beveridge (Eds.), *Children thinking through language.* (pp. 129-155). London.: Edward Arnold.

Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality.* London.: Routledge.

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